

The growth of matter perturbations in the $f(T)$ gravity

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Abstract

In this paper, we study the growth index of matter density perturbations for the power law model in $f(T)$ gravity. Using the parametrization $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{1+z}$ for the growth index, which approximates the real evolution of $\gamma(z)$ very well, and the observational data of the growth factor, we find that, at 1σ confidence level, the power law model in $f(T)$ gravity is consistent with the observations, since the obtained theoretical values of γ_0 and γ_1 are in the allowed region.

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I. INTRODUCTION

Cosmological data from a wide range of sources have indicated that our Universe is undergoing an accelerating expansion [1–3]. Basically, two kinds of alternative explanations have been proposed for this unexpected observational phenomenon. One is the dark energy with a sufficient negative pressure, which induces a late-time accelerating cosmic expansion. Currently, there are many candidates of dark energy, such as the cosmological constant, quintessence, phantom, quintom, and so on. The other is the modified gravity, which originates from the idea that the general relativity is incorrect in the cosmic scale and therefore needs to be modified. Examples of such theories are the scalar-tensor theory, the $f(R)$ theory and the Dvali-Gabadadze-Porrati (DGP) braneworld scenario, *et al.*

Recently, a new interesting modified gravity by extending the teleparallel theory [4], called $f(T)$ gravity, is proposed to explain the present accelerating cosmic expansion [5]. Since the teleparallel theory is based upon the Weitzenböck connection rather than the Levi-Civita one, it has no curvature but only torsion. In analogy to the well-known $f(R)$ gravity obtained from extending the Einstein-Hilbert action to be an arbitrary function of R , the $f(T)$ theory is built by generalizing the action of the teleparallel gravity to be $T + f(T)$ [5, 6]. An important advantage of the $f(T)$ gravity is that its field equations are second order as opposed to the fourth order equations of the $f(R)$ gravity. So, it has spurred an increasing deal of interest in the literatures [7–15]. For example, some concrete models are built, in [5, 6, 10, 11], to account for the present cosmic expansion and the models with the phantom divide line crossing are proposed in [10, 11]. The observational constraints on model parameters are discussed in [12] and the dynamical analysis for a general $f(T)$ theory is performed in [13].

It has been pointed out [6, 12] that the $f(T)$ theory can give the same background evolution as other models, such as the Λ CDM and DGP. So, in order to discriminate the $f(T)$ gravity from other models, one needs to break the degeneracy of the background expansion history. An interesting approach to differentiating the modified gravity and dark energy is to use the growth function $\delta(z) \equiv \delta\rho_m/\rho_m$ of the linear matter density

contrast [16–36]. While different models give the same late time expansion, they may produce different growth of matter perturbations [23]. To discriminate different models with the matter perturbation, usually, the growth factor $g \equiv \frac{d \ln \delta}{d \ln a}$ is used and it can be parameterized as [24, 25]

$$g \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m^\gamma, \quad (1)$$

where γ is the growth index and Ω_m is the fractional energy density of matter. This approach has been explored in many works [26–36], and it is found that different models may give different values of the growth index if a constant γ is considered. For example, $\gamma_\infty \simeq 0.5454$ [26, 28] for the Λ CDM model and $\gamma_\infty \approx 0.6875$ [26, 27] for the flat DGP model. Therefore, in principle, one can distinguish the modify gravity from dark energy with the observational data on the growth factor.

Recently, the authors in Refs. [14, 15] have discussed the matter perturbations in the $f(T)$ gravity and found that, despite the completely indistinguishable background behavior, the growth of matter density perturbation of different models can be different. In this paper, we plan to investigate the growth index of matter density perturbations in the $f(T)$ gravity, and then by using a feasible parametrization of the growth index to compare the growth function with the observational data. Our results show that the $f(T)$ gravity is consistent with the observations.

II. THE $f(T)$ THEORY

In this section, we give a brief review of the $f(T)$ gravity. The torsion scalar T in the action of the teleparallel gravity is defined as

$$T \equiv S_\sigma^{\mu\nu} T_{\mu\nu}^\sigma, \quad (2)$$

where

$$S_\sigma^{\mu\nu} \equiv \frac{1}{2} (K^{\mu\nu}_\sigma + \delta_\sigma^\mu T^{\alpha\nu}_\alpha - \delta_\sigma^\nu T^{\alpha\mu}_\alpha), \quad (3)$$

and $T^\sigma_{\mu\nu}$ is the torsion tensor

$$T^\sigma_{\mu\nu} \equiv e_A^\sigma (\partial_\mu e_\nu^A - \partial_\nu e_\mu^A). \quad (4)$$

Here e_μ^A is the orthonormal tetrad component, where A is an index running over 0, 1, 2, 3 for the tangent space of the manifold, while μ , also running over 0, 1, 2, 3, is the coordinate index on the manifold. $K^{\mu\nu}{}_\sigma$ is the contorsion tensor given by

$$K^{\mu\nu}{}_\sigma = -\frac{1}{2}(T^{\mu\nu}{}_\sigma - T^{\nu\mu}{}_\sigma - T_\sigma{}^{\mu\nu}) . \quad (5)$$

Similar to $f(R)$ gravity, the $f(T)$ theory is obtained by extending the action of teleparallel gravity to be $T + f(T)$ to explain the late time accelerating cosmic expansion with no need of an exotic dark energy.

Assuming a flat homogeneous and isotropic Friedmann-Robertson-Walker universe described by the metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j , \quad (6)$$

where a is the scale factor, one has, from Eq. (2),

$$T = -6H^2 , \quad (7)$$

with $H = \dot{a}/a$ being the Hubble parameter. Thus, in $f(T)$ gravity, the cosmic background equation can be written as

$$H^2 = \frac{8\pi G}{3}\rho - \frac{f}{6} - 2H^2 f_T , \quad (8)$$

$$\dot{H} = -\frac{1}{4} \frac{6H^2 + f + 12H^2 f_T}{1 + f_T - 12H^2 f_{TT}} , \quad (9)$$

with $f_T \equiv df/dT$. Here we assume that the energy component in our universe is only the matter with radiation neglected. Apparently, the last two terms in the right hand side of Eq. (8) can be regarded as an effective dark energy. Then, its effective energy density and equation of state can be expressed, respectively, as

$$\rho_{eff} = \frac{1}{16\pi G}(-f + 2T f_T) \quad (10)$$

$$w_{eff} = -\frac{f/T - f_T + 2T f_{TT}}{(1 + f_T + 2T f_{TT})(f/T - 2f_T)} . \quad (11)$$

In order to explain the present accelerating cosmic expansion, some $f(T)$ models are proposed in Refs. [5, 6, 10, 11]. In this paper, we only consider a power law model [5, 6]

$$f(T) = \alpha(-T)^n = \alpha(6H^2)^n, \quad (12)$$

with

$$\alpha = (6H_0^2)^{1-n} \frac{1 - \Omega_{m0}}{2n - 1}, \quad (13)$$

where $\Omega_{m0} = \frac{8\pi G \rho(0)}{3H_0^2}$ is the dimensionless matter density parameter today. Substituting above two expressions into Eq. (8) and defining $E^2 = H^2/H_0^2$, one has

$$E^2 = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})E^{2n}. \quad (14)$$

The reason to consider the power model in our paper is that it has the same background evolution equation as some phenomenological models [52, 53] and it reduces to the Λ CDM model when $n = 0$, and to the DGP model [54] when $n = 1/2$. In addition, it has a smaller χ_{Min}^2 value than the Λ CDM when fitting the recent observations such as the Type Ia Supernova (Sne Ia), the baryonic acoustic oscillation (BAO) and the Cosmic Microwave Background (CMB) radiation [12]. Let us note that, in order to be consistent with the present observational results, it is required that $|n| \ll 1$ [5, 6, 12].

III. GROWTH INDEX OF $f(T)$ MODEL

To the linear order of matter density perturbations, the growth function $\delta(z)$ at scale much smaller than the Hubble radius satisfies the following equation [24]

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff} \rho_m \delta = 0, \quad (15)$$

where G_{eff} is the effective Newton's constant and the dot denotes the derivative with respect to time t . In the general relativity, $G_{eff} = G_N$, where G_N is the Newton's constant. Defining the growth factor $g \equiv d \ln \delta / d \ln a$, one can obtain

$$\frac{d g}{d \ln a} + g^2 + \left(\frac{\dot{H}}{H^2} + 2 \right) g = \frac{3}{2} \frac{G_{eff}}{G_N} \Omega_m. \quad (16)$$

In general, the analytical solution of above equation is very difficult to obtain, and thus, we need to resort to the numerical methods.

In the $f(T)$ gravity, G_{eff} can be expressed as [15]

$$G_{eff} = \frac{G_N}{1 + f_T}. \quad (17)$$

So, the growth factor satisfies the following equation:

$$\frac{d g}{d \ln a} + g^2 + \left(\frac{\dot{H}}{H^2} + 2 \right) g = \frac{3}{2} \frac{1}{1 + f_T} \Omega_m. \quad (18)$$

Using

$$\frac{\dot{H}}{H^2} = -\frac{3}{2} \frac{1 + f/6H^2 + 2f_T}{1 + f_T - 12H^2 f_{TT}}, \quad (19)$$

and Eq. (13), we get

$$\frac{d g}{d \ln a} + g^2 + g \left[2 - \frac{3}{2} \frac{1 - E^{2n-2}(1 - \Omega_{m0})}{1 - nE^{2n-2}(1 - \Omega_{m0})} \right] = \frac{3}{2} \frac{\Omega_m}{1 - \frac{n(1 - \Omega_{m0})E^{2n-2}}{2n-1}}, \quad (20)$$

where $\Omega_m = \Omega_{m0} E^{-2} (1 + z)^{-3}$ and E satisfies

$$\frac{dE^2}{d \ln a} = \frac{-3E^2 + 3E^{2n}(1 - \Omega_{m0})}{1 - nE^{2n-2}(1 - \Omega_{m0})}. \quad (21)$$

Thus, given the values of Ω_{m0} and n , the value of g can be obtained by solving Eqs. (20) and (21) numerically with the initial condition $g = 1$ at $z \rightarrow \infty$. Then, using Eq. (1), we can get the value of growth index. The results are shown in Fig. (1). From the left panel of this figure, we can see that, when $n = 0$, our result reduces to that in the Λ CDM. As expected, the growth factor g in $f(T)$ gravity ($n \neq 0$) grows slower than that in general relativity, which is the same as that obtained in Refs. [14, 15], because a weak effective Newton gravity (see Eq. (17)) is obtained. The right panel shows that the growth index γ in $f(T)$ gravity is different from that in the Λ CDM at both low redshifts and high redshifts. This is caused by the fact that the $f(T)$ gravity and Λ CDM give different background evolutions at low redshifts. However, γ is mainly determined by the low redshift evolution, since, at high redshifts $\Omega_m \sim 1$, different values of γ may give the same result for g . When $\Omega_{m0} = 0.272$, the values of growth index γ approach to 0.58 and 0.71 with $z \rightarrow \infty$ for $n = 0.1$ and 0.25, respectively. These results are different from that obtained from the Λ CDM and DGP models where the values are $\gamma_\infty \approx 0.5454$ [26, 28] and

$\gamma_\infty \approx 0.6875$ [26, 27], respectively. This feature of γ_∞ provides a distinctive signature for $f(T)$ as opposed to the Λ CDM and DGP models. So, one can distinguish the $f(T)$ gravity from other cosmological models by the growth index of matter density perturbations.

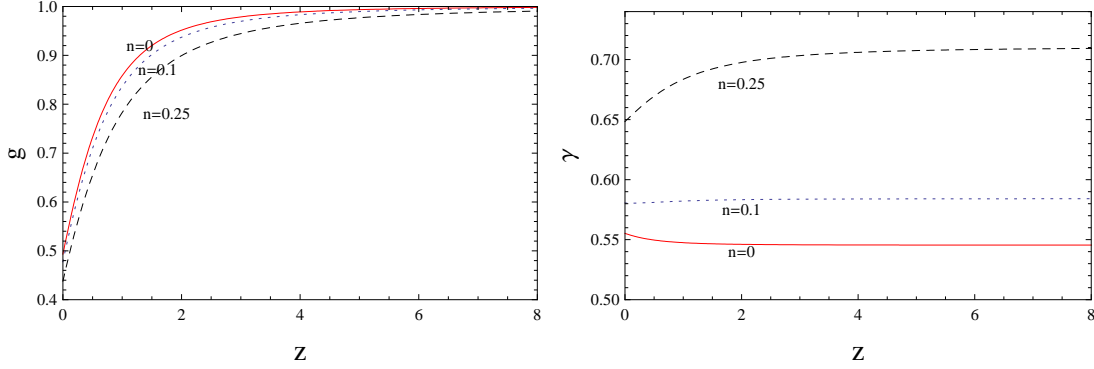


FIG. 1: while $\Omega_{m0} = 0.272$, g and γ are displayed as a function of z for $f(T)$ gravity respectively.

IV. GROWTH INDEX PARAMETRIZATION AND OBSERVATIONAL CONSTRAINTS

From the right panel of Fig. (1), it is easy to see that, for any n , γ is not a constant, especially in the redshifts region ($z < 2$) where some observational data points are obtained. So, it is unreliable to discriminate different models with these observational data if the growth index is treated as a constant. The growth index γ should be a function of z and we may parameterize it. Since Eq. (20) can be reexpressed as

$$\begin{aligned} & \frac{3\Omega_m(n-1)E^{2n-2}(1-\Omega_{m0})}{1-nE^{2n-2}(1-\Omega_{m0})} \frac{dg}{d\Omega_m} + g^2 + g \left[2 - \frac{3}{2} \frac{1-E^{2n-2}(1-\Omega_{m0})}{1-nE^{2n-2}(1-\Omega_{m0})} \right] \\ &= \frac{3}{2} \frac{\Omega_m}{1 - \frac{n(1-\Omega_{m0})E^{2n-2}}{2n-1}}, \end{aligned} \quad (22)$$

substituting Eq. (1) into the above expression, we obtain an equation of $\gamma(z)$

$$\begin{aligned} & -(1+z) \ln \Omega_m \gamma' + \Omega_m^\gamma + (2-3\gamma) - \frac{3}{2}(1-2\gamma) \frac{1-E^{2n-2}(1-\Omega_{m0})}{1-nE^{2n-2}(1-\Omega_{m0})} \\ &= \frac{3}{2} \frac{\Omega_m^{1-\gamma}}{1 - \frac{n(1-\Omega_{m0})E^{2n-2}}{2n-1}}. \end{aligned} \quad (23)$$

Here, we consider a parametrization form of $\gamma(z)$ proposed in Ref. [41]

$$\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{1+z}, \quad (24)$$

which gives a very good approximation of $\gamma(z)$ for the w CDM and DGP models. The error is below 0.03% for the Λ CDM model and 0.18% for the DGP model for all redshifts when $\Omega_{m0} = 0.27$. In $f(T)$ gravity, for any given values of n and Ω_{m0} , we can obtain the value of γ_0 through the value of g_0 determined from solving Eq. (22) numerically. Substituting Eq. (24) into Eq. (23), we can get the expression of γ_1 which is a function of redshifts z . For simplicity, we take the value of γ_1 at $z = 0$,

$$\gamma_1 = (\ln \Omega_{m0}^{-1})^{-1} \left[-\Omega_{m0}^{\gamma_0} + \frac{3}{2} \frac{\Omega_{m0}^{1-\gamma_0}}{1 - \frac{n(1-\Omega_{m0})}{2n-1}} - (2 - 3\gamma_0) + \frac{3}{2} \frac{\Omega_{m0}(1 - 2\gamma_0)}{1 - n(1 - \Omega_{m0})} \right]. \quad (25)$$

Thus, we obtain the possible regions of γ_0 and γ_1 for $0.20 \leq \Omega_{m0} \leq 0.35$. The results are shown in Fig. (2). We find, for a given value of Ω_{m0} , that the values of γ_0 and γ_1 in $f(T)$ gravity ($n \neq 0$) are larger than those in the Λ CDM model ($n = 0$), and a larger value of n gives larger values of γ_0 and γ_1 . The reason for these is that the effective gravity in $f(T)$ theory is weaker than that in general relative, and a larger n leads to a weaker effective gravity. These features provide distinctive signatures for $f(T)$ gravity as opposed to the Λ CDM model. Therefore, in principle, we can discriminate the $f(T)$ model from the Λ CDM model merely through the values of γ_0 and γ_1 if one can obtain their accurate values from the observation data.

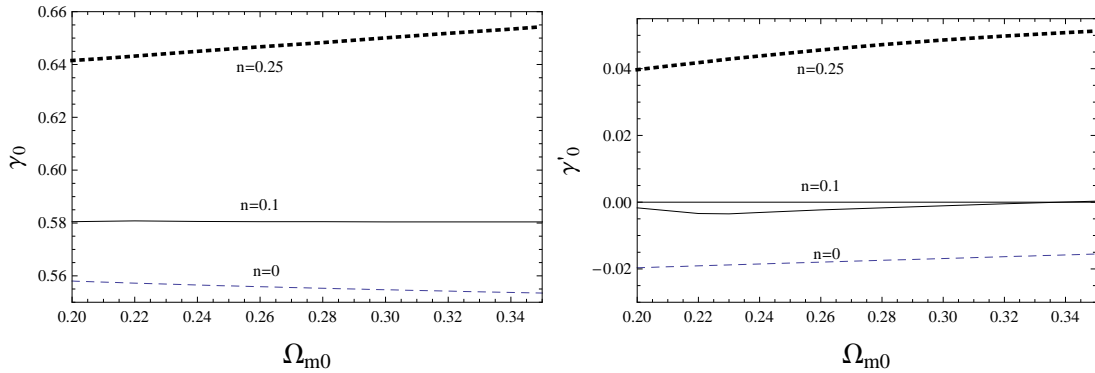


FIG. 2: γ_0 and γ_1 are displayed as a function of Ω_{m0} for $n = 0, 0.1, 0.25$ respectively.

Before studying the observational constraints on γ_0 and γ_1 , we need to examine how well the $\Omega_m^{\gamma(z)}$ with $\gamma(z)$ taking the parametrization in Eq. (24) approximates the growth factor g . Numerical results are shown in Fig. (3) with $\Omega_{m0} = 0.272$. From this figure, we see that the error is below 0.45% for $n = 0.25$ and below 0.1% for $n = 0.1$. So, $\Omega_m^{\gamma_0 + \gamma_1 \frac{z}{1+z}}$ approximates the growth factor g very well both at low and high redshifts in $f(T)$ theory, and we can use all data points to constrain this parametrization.

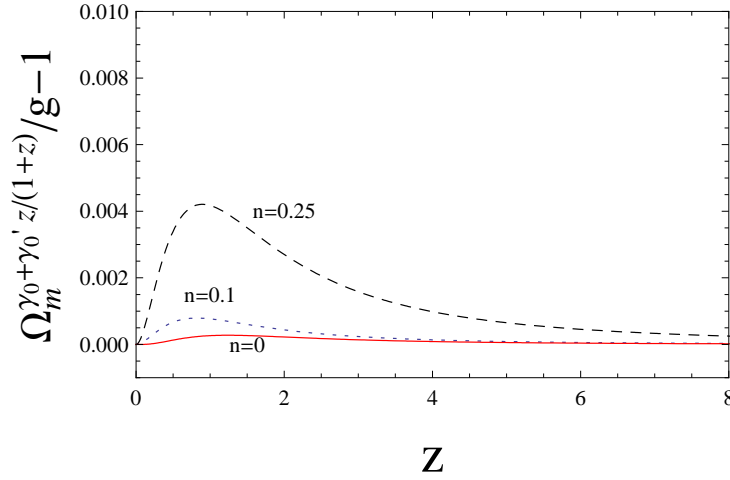


FIG. 3: The relative difference between the growth factor g and Ω_m^γ with $\gamma = \gamma_0 + \gamma_1 \frac{z}{1+z}$ and $\Omega_{m0} = 0.272$. The dashed, dotted and solid curves show the results of $n = 0.25, 0.1, 0$, respectively.

In order to obtain the observational constraints on γ_0 and γ_1 , we first need to determine the value of Ω_{m0} and n from the observations. Here we use the results obtained from the combination of the latest Union2 Type Ia Supernova (Sne Ia) set, the BAO from the SDSS data and the Cosmic Microwave Background (CMB) radiation [12]. At the 95% confidence level, $\Omega_{m0} = 0.272^{+0.036}_{-0.032}$, $n = 0.04^{+0.22}_{-0.33}$ for the power law model. With these best fit values and Eqs. (20, 25), we obtain that the corresponding theoretical values of γ_0 and γ_1 are $\gamma_0 = 0.564$ and $\gamma_1 = -0.0123$.

Now we discuss the observational constraints on γ_0 and γ_1 from the growth factor data. Here 12 data points given in Table I are used. Let us note that although the data

given in Refs. [42, 43] are measured without ‘any’ bias, other data points are obtained by assuming a flat Λ CDM model with Ω_{m0} taking a specific value, for example, $\Omega_{m0} = 0.25$ or 0.30. So, caution must be exercised when using these data. With this caveat in mind, it may still be worthwhile to apply the data to fit the models [27, 44, 45]. With the best fit values of Ω_{m0} and n , we can obtain the constraints on γ_0 and γ_1 from the observations by using the following equation

$$\chi_g^2 = \sum_{i=1}^{12} \frac{[g_{obs}(z_i) - \Omega_m^{\gamma_0 + \gamma_1 z_i / (1+z_i)}]^2}{\sigma_{gi}^2}, \quad (26)$$

where σ_{gi} is the 1σ uncertainty of the $g(z_i)$ data. We find that the best fit values are $\gamma_0 = 0.809$ and $\gamma_1 = -0.942$. The allowed regions at 1 and 2σ confidence levels are shown in Fig.(4), from which, one can see that the power law model in $f(T)$ gravity is consistent with the observations, since the theoretical values of γ_0 and γ_1 obtained by using the best fit values of Ω_{m0} and n for the power law model are in the allowed region at 1σ confidence level.

z	g_{obs}	References
0.15	0.49 ± 0.1	[46, 47]
0.35	0.7 ± 0.18	[48]
0.55	0.75 ± 0.18	[49]
0.77	0.91 ± 0.36	[46, 47]
1.4	0.9 ± 0.24	[50]
3.0	1.46 ± 0.29	[51]
2.125 – 2.72	0.74 ± 0.24	[42]
2.2 – 3	0.99 ± 1.16	[43]
2.4 – 3.2	1.13 ± 1.07	[43]
2.6 – 3.4	1.66 ± 1.35	[43]
2.8 – 3.6	1.43 ± 1.34	[43]
3 – 3.8	1.3 ± 1.5	[43]

TABLE I: The summary of the observational data on the growth factor g .

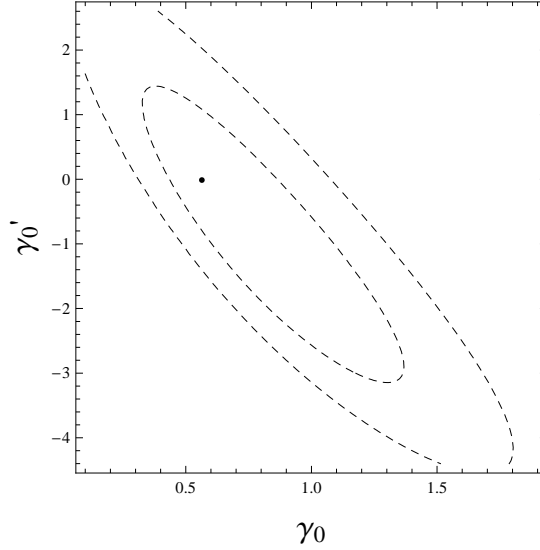


FIG. 4: The 1σ and 2σ contours of γ_0 and γ_1 by fitting the power law model in $f(T)$ gravity to the growth rate data. The point denotes the theoretical values of γ_0 and γ_1 with the Ω_{m0}, n taking the best fit values.

V. CONCLUSION

In this paper, we study in detail the growth index of matter density perturbations for the power law model in $f(T)$ gravity. Using a parametrization $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{1+z}$ for the growth index $\gamma(z)$, which gives a very good approximation of $\gamma(z)$, we find that the value of γ_0 and γ_1 in $f(T)$ gravity are larger than those in the Λ CDM model, and a larger value of n gives larger values of γ_0 and γ_1 . This feature may provide a signature for $f(T)$ gravity distinctive from the other models, such as the Λ CDM and DGP. Finally, we discuss the constraints on γ_0 and γ_1 from the observational growth factor data and find that, at 1σ confidence level, the power law model in $f(T)$ gravity is consistent with the observations since the theoretical values of γ_0 and γ_1 obtained by using the best fit values of Ω_{m0} and n are in the allowed region.

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- [1] A. G. Riess, *et al.*, *Astron. J.* **116**, 1009 (1998); S. J. Perlmutter, *et al.*, *Astrophys. J.* **517**, 565 (1999).
 - [2] D. N. Spergel, *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003); D. N. Spergel, *et al.*, *Astrophys. J. Suppl.* **170**, 377S (2007).
 - [3] D. J. Eisenstein *et al.*, *Astrophys. J.* **633**, 560 (2005); E. Komatsu *et al.*, *Astrophys. J. Suppl.* **180**, 330 (2009) .
 - [4] A. Einstein, *Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl.*, 217 (1928); 401 (1930); A. Einstein, *Math. Ann.* **102**, 685 (1930); K. Hayashi and T. Shirafuji, *Phys. Rev. D* **19**, 3524 (1979); **24**, 3312 (1981).
 - [5] G. R. Bengochea and R. Ferraro, *Phys. Rev. D* **79**, 124019 (2009).
 - [6] E. V. Linder, *Phys. Rev. D* **81**, 127301 (2010).
 - [7] K. K. Yerzhanov, S. R. Myrzakul, I. I. Kulnazarov, R. Myrzakulov, arXiv:1006.3879; R. Yang, arXiv:1007.3571; P. Yu. Tsyba, I. I. Kulnazarov, K. K. Yerzhanov, R. Myrzakulov, arXiv:1008.0779; S. Chen, J. B. Dent, S. Dutta, E. N. Saridakis, arXiv:1008.1250; G. R. Bengochea, arXiv:1008.3188; K. Bamba, C. Geng, C. Lee, arXiv:1008.4036; R. Myrzakulov, arXiv:1008.4486; K. Karami, A. Abdolmaleki, arXiv:1009.2459; K. Karami, A. Abdolmaleki, arXiv:1009.3587; B. Li, T. P. Sotiriou, J. D. Barrow, arXiv:1010.1041; T. P. Sotiriou, B. Li, J. D. Barrow, arXiv:1012.4039; R. Ferraro, F. Fiorini, arXiv:1103.0824; Y. Zhang, H. Li, Y. Gong, Z. Zhu, arXiv:1103.0719; T. Wang, arXiv:1102.4410; R. Ferraro, F. Fiorini, arXiv:1103.0824; B. Li, T. P. Sotiriou, J. D. Barrow, arXiv:1103.2786.
 - [8] R. Ferraro and F. Fiorini, *Phys. Rev. D* **75**, 084031 (2007).

- [9] R. Ferraro and F. Fiorini, Phys. Rev. D **78**, 124019 (2008).
- [10] K. Bamba, C. Geng, C. Lee, L. Luo, JCAP **1101**, 021 (2011).
- [11] P. Wu and H. Yu, Eur. Phys. J. C **71**, 1552 (2011).
- [12] P. Wu and H. Yu, Phys. Lett. B **693**, 415 (2010).
- [13] P. Wu and H. Yu, Phys. Lett. B **692**, 176 (2010).
- [14] S. Dutta and E. N. Saridakis, JCAP **1101**, 009 (2011).
- [15] R. Zheng and Q. Huang, JCAP **1103**, 002 (2011).
- [16] X. Fu, P. Wu and H. Yu, Eur. Phys. J. C **68**, 271 (2010).
- [17] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. **95**, 141301 (2005); E. V. Linder, Phys. Rev. D **73**, 063010 (2006).
- [18] R. J. Scherrer, Phys. Rev. D **73**, 043502 (2006).
- [19] T. Chiba, Phys. Rev. D **73**, 063501 (2006).
- [20] E. V. Linder, Gen. Rel. Grav. **40**, 329 (2008).
- [21] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. **77**, 201 (2003);
U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. **344**, 1057 (2003).
- [22] Y. Gong, M. Ishak and A. Wang, Phys. Rev. D **80**, 023002 (2009).
- [23] A. A. Starobinsky, JETP Lett. **68**, 757 (1998).
- [24] L. M. Wang and P. J. Steinhardt, Astrophys. J. **508**, 483 (1998).
- [25] J. N. Fry, Phys. Lett. B **158**, 211 (1985).
A. P. Lightman and P.L. Schechter, Astrophys. J. **74**, 831 (1990) .
- [26] E. V. Linder and R. N. Cahn, Astropart. Phys. **28**, 481 (2007).
- [27] H. Wei, Phys. Lett. B **664**, 1 (2008).
- [28] E. V. Linder, Phys. Rev. D **72**, 043529 (2005).
- [29] D. Huterer and E. V. Linder, Phys. Rev. D **75**, 023519 (2007).
- [30] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D **69**, 124015 (2004).
- [31] A. Lue, Phys. Rept. **423**, 1 (2006).
- [32] C. Di Porto and L. Amendola, Phys. Rev. D **77**, 083508 (2008);
L. Amendola, M. Kunz and D. Sapone, JCAP **0804**, 013 (2008).

- [33] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **77**, 023504 (2008).
- [34] Y. Wang, JCAP **5**, 21 (2008).
- [35] B. Boisseau, G. Esposito-Farèse, D. Polarski, A.A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000).
- [36] Y. Gong, Phys. Rev. D **78**, 123010 (2008).
- [37] D. Polarski and R. Gannouji, Phys. Lett. B **660**, 439 (2008).
- [38] R. Gannouji and D. Polarski, JCAP **0805**, 018 (2008).
- [39] R. Gannouji, B. Moraes, D. Polarski, arXiv: 0809.3374.
- [40] X. Fu, P. Wu and H. Yu, Phys. Lett. B **677**, 12 (2009).
- [41] P. Wu, H. Yu and X. Fu, JCAP **0906**, 019 (2009).
- [42] M. Viel , M. G. Haehnelt and V. Springel, Mont. Not. R. Astron. Soc. **354**, 684 (2004).
- [43] M. Viel, M. G. Haehnelt and V. Springel, Mont. Not. R. Astron. Soc. **365**, 231 2006.
- [44] Y. Gong Phys. Rev. D **78**, 123010 (2008).
- [45] Y. Gong M. Ishak and A. Wang Phys. Rev. D **80**, 023002 (2009) .
- [46] L. Guzzo, M. Pierleoni *et al.* Nature **451**, 541 (2008) .
- [47] M. Colless, *et al.* Mont. Not. R. Astron. Soc. **328**, 1039 (2001).
- [48] M. Tegmark, *et al.* Phys. Rev. D **74**, 123507 (2006) .
- [49] N. P. Ross, *et al.* Mont. Not. R. Astron. Soc. **381**, 573 (2007).
- [50] J. da Ângela , *et al.* Mont. Not. R. Astron. Soc. **383**, 565 (2008) .
- [51] P. McDonald, *et al.* Astrophys. J. **635**, 761 (2005) .
- [52] G. Dvali and M. S. Turner, arXiv: astro-ph/0301510.
- [53] D. J. H. Chung and K. Freese, Phys. Rev. D **61**, 023511 (1999).
- [54] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000).